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A mathematical model for the determination of leakage in mains and water distribution networks

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Abstract. The inverse problem of decomposing uncontrolled water flow to the unregistered consumption and the leakage in mains and water distribution networks is considered. A mathematical model is proposed for the determination of unregistered consumption and leakage using the heads and flows at the inlet and at the outlet of the main or at some nodes of the network. The cases of discretely and continuously distributed water consumption in mains are thoroughly analysed and equations for the determination of respective parameters derived.

Key words: water distribution system, leakage, inverse problem, mathematical model.

1. INTRODUCTION

Water distribution systems are commonly examined using mathematical models, which describe their hydraulic qualities. To represents the physical system correctly, the network model must be calibrated. As the first step in the calibration procedure, uncontrolled flows in the network must be studied. The uncontrolled flows are due to leaks, illegal connections, meter errors, etc. Usually available data for the estimation of volumes and spatial and time distribution of uncontrolled flows is insufficient. Therefore the correction parameters of the mathematical model can not be obtained directly. Usually for the determination of model parameters the measured pressure and flow data are used.

Recently some methods have been proposed for the evaluation of the distribution of leaks in water distribution networks. It has been recommended to simulate the leaks with a fictitious discharge valve and to determine the number of defects by statistical criteria and available recorded data [1]. Based on an empirical expression, a formula, which relates the leakage flow to pressure with the power of 1.18 is proposed in [2]. A formula for water losses, which in addition to pressure consider leak's surface, is given in [3]. An algorithm, which distributes leakage between the nodes proportionally to the pressure in the nodes, is proposed in [4]. In this model, an iteration process, which determines the leakage and pressure intermittently, was applied. In [5], a simulation model is proposed in which leaks are allocated at nodes and the related flows depend on the pressure with the power of 0.5 for the burst and of 1.5 for the background leakage. An intensively metered study of leakage and demand on the district meter area is described in [6]. Statistical analysis is used for decomposing the flows into constituent components of domestic, commercial and pressure-related leakage. In [7] the problem of adequate representation of real demand and losses and their dependence on water pressure is considered.

Recently leak detection methodologies have been worked out, which are based on transient analysis. To estimate leak location and size, the data from transient tests are analysed either in the time domain or in the frequency domain. Within the first approach, the real time values of pressure and flow at the pipe end [8,9] or only the pressure–time history in one section are used [10]. Within the second approach, the analysis of the Fourier transform of the pressure signal is used and compared with the experimental transfer function [11–13].

The pattern, which the presence of a leak imposes on the resonance peaks of the frequency response diagram, has also been considered [14]. In [15], a fuzzy system is used for detecting leaks in the cases when operational or process transients are generated in the system, which incorporate the uncertainty aspect. The Lyapunov stability criteria is used for leak detection in a continuously monitored pipeline in [16].

The uncontrolled flow can be divided into two parts: the unregistered consumption, which consists mainly of illegal connections and water meter errors, and leakage. It can be assumed that the flow of the first part is distributed similarly to registered consumption. This permits to decompose the given flow into two parts: consumption, in which distribution is known, and leakage, in which distribution depends on the pressure. In [17] a methodology for the evaluation of water losses in a water distribution network is given. This methodology disregards the existence of the two components of uncontrolled water: physical losses in mains and the volume of water, consumed but not measured by meters.

In this paper we consider the problem of decomposing the flow between the consumption and leakage. An inverse parametric problem for the determination of the decomposition of uncontrolled flows in mains and water distribution networks is formulated and solved.

2. MAINS WITH DISCRETELY DISTRIBUTED WATER CONSUMPTION

Let us analyse the leakage in a main with a simple mathematical model. Consider a main with discretely distributed registered water consumptions Q_i, unregistered consumptions Q_i^*, and leakages $\tilde{Q}_i$ at points x_i $(i = 1, 2, \ldots, n)$ (Fig. 1).

Denote by Q_0 and Q_L the volumes of the flow at the inlet and at the outlet of the main. Assume that $Q_i^* = (m-1)Q_i$ so that

$$Q_i^* + Q_i = mQ_i. \tag{1}$$

Here $1/m$ is the registration coefficient of consumption. Assume that $m > 1$.

Denote by $H(x)$ the total head and by $q(x)$ the flow in the main at the point x. Let $H_0 = H(0)$, $H_i = H(x_i)$ and $H_L = H(L)$. Let the head loss in the interval $[x_i, x_{i+1}]$ be

$$h_i = cl_i q_i^{\alpha}, \tag{2}$$

where c is the resistance coefficient, $l_i = x_{i+1} - x_i$, q_i is the flow in the interval $[x_{i+1}, x_i]$ and α is the flow exponent.

Since

$$q_0 = Q_0,$$

$$q_i = Q_0 - \sum_{k=1}^{i} (mQ_k + \tilde{Q}_k), \quad i = 1, 2, \ldots, n,$$

$$q_n = Q_n \tag{3}$$

we have

$$H_1 = H_0 - cl_0 Q_0^{\alpha},$$

$$H_{i+1} = H_i + cl_i \left[Q_0 - \sum_{k=1}^{i} (mQ_k + \tilde{Q}_k) \right]^{\alpha}, \quad i = 1, 2, ..., n,$$

$$H_{n+1} = H_L. \tag{4}$$

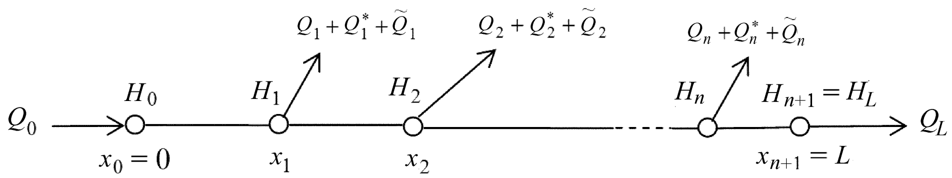

Fig. 1. Main with discretely distributed water consumption.

Assume that the local leakages at the points x_i can be expressed as

$$\tilde{Q}_i = kH_i^{\beta}, \quad i = 1, 2, ..., n, \tag{5}$$

where k is a coefficient, which characterizes the leakage intensity and β is the loss exponent.

Let us now consider the following inverse problem for flows in the main. Assume that the flows Q_0, Q_i, Q_L and heads H_0 and H_L are given. From these data the leakages $\tilde{Q}_i$ and unregistered consumptions Q_i^* are to be found. For this the coefficients m and k, and heads H_i must be determined. After that Q_i^* and $\tilde{Q}_i$ can be obtained from Eqs. (1) and (5).

For the determination of $n+2$ unknowns H_i, m and k we can use the following equations. The mass conservation law gives

$$\sum_{i=1}^{n} (mQ_i + kH_i^{\beta}) = Q_0 - Q_L. \tag{6}$$

From Eq. (3), for energy conservation we have

$$\begin{gathered} H_1 = H_0 - cl_0 Q_0^{\alpha}, \\ H_{i+1} = H_i - cl_i \left[Q_0 - \sum_{s=1}^{i} (mQ_s + kH_s^{\beta}) \right]^{\alpha}, \quad i = 1, 2, ..., n, \\ H_{n+1} = H_L. \end{gathered} \tag{7}$$

By elimination of the heads H_i, the system (6), (7) can be reduced to two equations for the determination of m and k.

Now, let us consider a simple special case when Eqs. (5) and (6) can be easily solved. Assume that $\alpha = 2$ and $\beta = 1$. Then for $n = 2$ from Eqs. (6) and (7) we obtain

$$\begin{gathered} m(Q_1 + Q_2) + k(H_1 + H_2) = Q_0 - Q_L, \\ H_1 = H_0 - cl_0 Q_0^2, \\ H_2 = H_1 - cl_1 [Q_0 - mQ_1 - kH_1]^2, \\ H_L = H_2 - cl_2 [Q_0 - mQ_1 - kH_1 - mQ_2 - kH_2]^2. \end{gathered} \tag{8}$$

The solution of this system can be expressed as

$$\begin{gathered} m = \frac{H_2(Q_0 - R) - H_1(R - Q_2)}{Q_1 H_2 - Q_2 H_1}, \\ k = \frac{Q_1(R - Q_L) - Q_2(Q_0 - R)}{Q_1 H_2 - Q_2 H_1}, \end{gathered} \tag{9}$$

where

$$H_1 = H_0 - cl_0 Q_0^2,$$
$$H_2 = H_L + cl_2 Q_L^2, \tag{10}$$

and

$$R = \sqrt{\frac{H_1 - H_2}{cl_1}}. \tag{11}$$

For the cases $n > 2$, system of equations (6), (7) reduces to the solution of an algebraic system of equations, which contains unknown parameters m and k in the power of $2(n-1)$.

3. MAINS WITH CONTINUOUSLY DISTRIBUTED WATER CONSUMPTION

Consider now a main with continuously distributed registered consumption $Q(x)$, unregistered consumption $Q^*(x)$, leakage $\tilde{Q}(x)$ and head $H(x)$ (Fig. 2).

Denote by $h(x)$ the head loss due to the pipe friction in the interval $(0, x)$ of the pipe, by H_0 and Q_0 the head and flow at the entrance of the pipe, and by H_L and Q_L the head and flow at the outlet of the pipe. Assume that $Q^*(x) = (m-1)Q(x)$, i.e.

$$Q^*(x) + Q(x) = mQ(x). \tag{12}$$

The head and the flow in the pipe at the point x are

$$H(x) = H_0 - h(x), \tag{13}$$

$$q(x) = Q_0 - \int_0^x [mQ(x) + \tilde{Q}(x)]\,\mathrm{d}x. \tag{14}$$

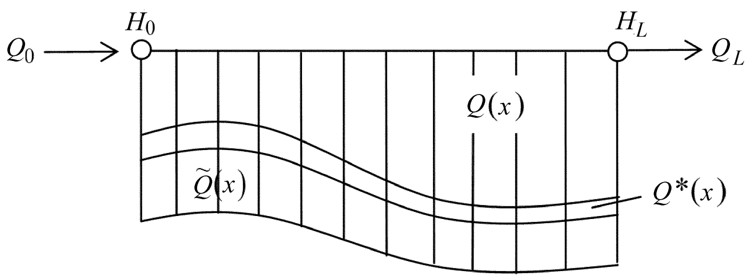

Fig. 2. Main with continuously distributed water consumption.

The head loss can be expressed as

$$h(x) = c\int_0^x [q(x)]^{\alpha}\,\mathrm{d}x, \tag{15}$$

or

$$h(x) = c\int_0^x \left\{ Q_0 - \int_0^x [mQ(x) + \tilde{Q}(x)]\,\mathrm{d}x \right\}^{\alpha} \mathrm{d}x, \tag{16}$$

where c is the resistance coefficient and α is the flow exponent.

Assume that

$$\tilde{Q}(x) = k[H(x)]^{\beta}, \tag{17}$$

where k is the leakage intensity coefficient and β is the loss exponent.

From Eqs. (13)–(17) follows

$$H(x) = H_0 - c\int_0^x \left\langle Q_0 - \int_0^x \{mQ(x) + k[H(x)]^{\beta}\}\,\mathrm{d}x \right\rangle^{\alpha} \mathrm{d}x. \tag{18}$$

With differentiation of Eq. (18) we obtain

$$\frac{\mathrm{d}H}{\mathrm{d}x} = -c\left\langle Q_0 - \int_0^x \{mQ(x) + k[H(x)]^{\beta}\}\,\mathrm{d}x \right\rangle^{\alpha}, \tag{19}$$

and

$$\frac{\mathrm{d}^2H}{\mathrm{d}x^2} = \alpha c\left\langle Q_0 - \int_0^x \{mQ(x) + k[H(x)]^{\beta}\}\,\mathrm{d}x \right\rangle^{\alpha-1} \cdot \{mQ(x) + k[H(x)]^{\beta}\}. \tag{20}$$

Substituting Eq. (19) into (20), we have

$$\frac{\mathrm{d}^2H}{\mathrm{d}x^2} - \alpha c\left(-\frac{\mathrm{d}H}{\mathrm{d}x} \right)^{\frac{\alpha-1}{\alpha}} \{mQ(x) + k[H(x)]^{\beta}\} = 0. \tag{21}$$

The boundary conditions for Eq. (21) follow from Eqs. (18) and (19) as

$$H(0) = H_0, \quad \left.\frac{\mathrm{d}H}{\mathrm{d}x}\right|_{x=0} = -cQ_0^{\alpha}. \tag{22}$$

From Eqs. (21) and (22) the head is determined in the form of $H(x; m, k)$. Now the inverse problem for the determination of the coefficients m and k can be solved. For that the equations

$$H(L; m, k) = H_L, \tag{23}$$

$$\int_0^L \{mQ(x) + k[H(x; m, k)]^\beta\} \mathrm{d}x = Q_0 - Q_L, \tag{24}$$

can be used.

Since the solution of the non-linear differential equation (21) in the general case cannot be expressed through analytic functions, we shall further consider some simplified versions of this equation.

4. MAINS WITH CONSTANT CONSUMPTION

Let us consider a special case of continuously distributed consumption when $\alpha = 2$, $\beta = 1$ and $Q(x) \cong 1$, $Q^*(x) = m - 1$.

For this case Eq. (21) has the following form

$$\frac{\mathrm{d}^2 H}{\mathrm{d}x^2} + 2\sqrt{c}\,k\sqrt{-\frac{\mathrm{d}H}{\mathrm{d}x}}\left[\frac{m}{k} + H(x)\right] = 0. \tag{25}$$

Taking

$$x = x(H), \quad p = \frac{\mathrm{d}H}{\mathrm{d}x}, \quad p = p[x(H)], \tag{26}$$

we have

$$\frac{\mathrm{d}^2 H}{\mathrm{d}x^2} = \frac{\mathrm{d}p}{\mathrm{d}H}\, p(H). \tag{27}$$

Respectively Eq. (25) can be written as

$$\frac{\mathrm{d}p}{\mathrm{d}H}\, p(H) + 2\sqrt{c}\,k\sqrt{-p(H)}\left(\frac{m}{k} + H\right) = 0. \tag{28}$$

The solution of Eq. (28) with the boundary condition

$$p(H_0) = -cQ_0^2 \tag{29}$$

has the form

$$p = -\left[3\sqrt{c}\left(mH + \frac{1}{2}kH^2\right) + \frac{3}{2}c_1\right]^{2/3}, \tag{30}$$

where

$$c_1 = \frac{2}{3}(cQ_0^2)^{3/2} - 2\sqrt{c}\left(mH_0 + \frac{1}{2}kH_0^2\right). \tag{31}$$

Therefore

$$\frac{\mathrm{d}H}{\mathrm{d}x} = -\left[3\sqrt{c}\left(mH + \frac{1}{2}kH^2\right) + \frac{3}{2}c_1\right]^{2/3}. \tag{32}$$

The solution of Eq. (32) with the boundary condition (22) can be written as

$$H(x) = \sqrt{\frac{R^2 S(x)}{1 - S(x)}} - \frac{m}{k}, \tag{33}$$

where

$$R^2 = \frac{c_1}{2\sqrt{c}k} - \frac{m^2}{k^2}, \tag{34}$$

and

$$S(x) = \frac{H_0 - \frac{m}{k}}{\sqrt{\left(H_0 + \frac{m}{k}\right)^2 + R^2}} - \left(\sqrt{\frac{3}{2}\sqrt{c}k}\right)^3 R^2 x. \tag{35}$$

Now, to solve the inverse problem and to determine the coefficients m and k, Eqs. (23), (24) and (33) must be used. From these equations we obtain

$$\sqrt{\frac{R^2 S(L)}{1 - S(L)}} - \frac{m}{k} = H_L, \tag{36}$$

and

$$k\int_0^L \sqrt{\frac{R^2 S(x)}{1 - S(x)}}\mathrm{d}x = Q_0 - Q_L - mL. \tag{37}$$

5. MAINS WITH SMALL LEAKAGE

Consider now the leakage problem for a special case when the leakage is relatively small. Let $\alpha = 2$, $\beta = 1$ and assume that $k \ll 1$. If we consider the constant k as a small parameter, then we can solve the problem (21) and (22) with the perturbation method.

Take

$$H(x) = H^{(0)}(x) + kH^{(1)}(x) + k^2 H^{(2)}(x) + \dots . \tag{38}$$

Substituting Eq. (38) into Eqs. (21) and (22), we obtain

$$\frac{\mathrm{d}^2 H^{(0)}}{\mathrm{d}x^2} - 2\sqrt{c}\, mQ(x)\sqrt{-\frac{\mathrm{d}H^{(0)}}{\mathrm{d}x}} = 0, \tag{39}$$

$$\frac{\mathrm{d}^2 H^{(1)}}{\mathrm{d}x^2} - \sqrt{c}\, mQ(x)\frac{1}{\sqrt{-\frac{\mathrm{d}H^{(0)}}{\mathrm{d}x}}}\frac{\mathrm{d}H^{(1)}}{\mathrm{d}x} - 2\sqrt{c}\sqrt{-\frac{\mathrm{d}H^{(0)}}{\mathrm{d}x}}H^{(0)} = 0, \tag{40}$$

....

and the following boundary conditions:

$$H^{(0)}(0) = H_0, \quad \left.\frac{\mathrm{d}H^{(0)}}{\mathrm{d}x}\right|_{x=0} = -cQ_0^2, \tag{41}$$

$$H^{(1)}(0) = 0, \quad \left.\frac{\mathrm{d}H^{(1)}}{\mathrm{d}x}\right|_{x=0} = 0, \tag{42}$$

....

The solution of Eq. (39) with the boundary conditions (41) can be written as

$$H^{(0)}(x) = H_0 - \int_0^x [q(x)]^2 \mathrm{d}x, \tag{43}$$

where

$$q(x) = Q_0 - m\int_0^x Q(x)\,\mathrm{d}x. \tag{44}$$

Now Eq. (40) takes the form

$$\frac{\mathrm{d}^2 H^{(1)}}{\mathrm{d}x^2} + \frac{\frac{\mathrm{d}q}{\mathrm{d}x}}{q(x)} \frac{\mathrm{d}H^{(1)}}{\mathrm{d}x} - K(x) = 0, \tag{45}$$

where

$$K(x) = 2cq(x)\left[H_0 - c\int_0^x [q(x)]^2 \,\mathrm{d}x \right]. \tag{46}$$

Solution of Eqs. (42) and (45) can be written as

$$H^{(1)}(x) = \int_0^x \frac{1}{q(x)} \int_0^x q(x) K(x) \,\mathrm{d}x. \tag{47}$$

Thus in the first approximation we have

$$H(x) = H_0 - \int_0^x [q(x)]^2 \mathrm{d}x + k\int_0^x \frac{1}{q(x)} K(x) \,\mathrm{d}x, \tag{48}$$

and for the determination of the leakage we have the formula

$$\tilde{Q}(x) = k\left[H_0 - \int_0^x [q(x)]^2 \,\mathrm{d}x + k\int_0^x \frac{1}{q(x)} K(x) \,\mathrm{d}x \right]. \tag{49}$$

Now, using Eqs. (48) and (49), the inverse problem for the determination of the coefficients m and k can be solved. For that the following equations must be used:

$$H(L) = H_L, \tag{50}$$

$$\int_0^L [mQ(x) + \tilde{Q}(x)] \,\mathrm{d}x = Q_0 - Q_L. \tag{51}$$

Thus if the water consumption $Q(x)$ and the constants Q_0, Q_L, H_0 and H_L are known, the leakage and unregistered water consumption can be determined.

6. WATER DISTRIBUTION NETWORK

Consider now the same problem for a water distribution network. Let the network have p pipes, l loops, one fixed head node (inlet) and n unknown head nodes (outlets). For such a network the following identity holds

$$p = n + l. \tag{52}$$

Assume that the topology of the network is given by the following incidence matrices.

The unknown head nodes incidence $(p \times n)$ matrix is

$$A = [a_{ij}], \tag{53}$$

where

$$a_{ij} = \begin{cases} 1, & \text{if the flow of pipe } i \text{ enters node } j, \\ 0, & \text{if pipe } i \text{ is not connected with node } j, \\ -1, & \text{if the flow of pipe } i \text{ leaves node } j. \end{cases}$$

The fixed head node incidence $(p \times l)$ matrix is

$$B = [b_i], \tag{54}$$

where

$$b_i = \begin{cases} 1, & \text{if the flow of pipe } i \text{ comes from the fixed head node}, \\ 0, & \text{if pipe } i \text{ is not connected with the fixed head node}. \end{cases}$$

The unknown nodal heads are defined by $(1 \times n)$ vector

$$H^T = [H_1, H_2, \ldots, H_n]. \tag{55}$$

Let the assigned nodal demands be given by the $(1 \times n)$ vector

$$Q^T = [Q_1, Q_2, \ldots, Q_n] \tag{56}$$

and the leakages by the $(1 \times n)$ vector

$$\tilde{Q}^T = [\tilde{Q}_1, \tilde{Q}_2, \ldots, \tilde{Q}_n]. \tag{57}$$

We assume that

$$\tilde{Q} = mQ + kH. \tag{58}$$

The unknown pipe flows are defined by the $(1 \times p)$ vector

$$q^T = [q_1, q_2, \ldots, q_p]. \tag{59}$$

The head loss $(1 \times p)$ vector can be expressed as

$$h = Dq, \tag{60}$$

where D is the hydraulic impedance matrix in the form

$$D = \begin{bmatrix} c_1 \left|q_1\right|^{\alpha-1} & & & \\ & c_2 \left|q_2\right|^{\alpha-1} & & \\ & & \cdots & \\ & & & c_p \left|q_p\right|^{\alpha-1} \end{bmatrix}. \tag{61}$$

The unknown head and flow vector components H_i and q_k, and coefficients m and k are determined from the following energy and mass conservation laws:

$$AH + Dq = -BH_0, \tag{62}$$

$$A^T q = mQ + kH, \tag{63}$$

$$-Bq^T = Q_0, \tag{64}$$

and complementary equations

$$Q_0 = m\sum_{i=1}^{n} Q_i + k\sum_{i=1}^{n} H_i, \tag{65}$$

$$H_s(m, k) = H_L. \tag{66}$$

Here H_L is assigned head in some nodes of the network and H_0 and Q_0 denote the assigned nodal head and flow at the inlet node. Solution of the system (62)–(66) can be obtained with software packages for the modelling of water distribution networks (for example EPANET) using iterative technique.

7. CONCLUSIONS

A mathematical model for the determination of the leakage and unregistered water consumption in mains and in water distribution networks is proposed. It is assumed that the water consumption distribution is known and the heads and flows at the inlet and at the outlet of the main or at some nodes of the network are given. The cases of discretely and continuously distributed consumption are analysed and equations for the determination of respective parameters derived.

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Peatorustiku ja veevõrgu lekete määramise matemaatiline mudel

Tiit Koppel, Leo Ainola ja Raido Puust

On loodud matemaatiline mudel, mis võimaldab määrata registreerimata tarbimised ja lekked, lähtudes survest ning vooluhulgast kas peatoru alguses ja lõpus või veevõrgu sõlmpunktides. Põhjalikult on analüüsitud nii diskreetse kui ka pideva jaotusega veetarbimise juhtu ja esitatud võrrandid veevõrgu parameetrite määramiseks.