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Separation of Lamb modes at a plate edge by using orthogonality relation

Madis Ratassepp[a], Aleksander Klauson[a], Farid Chati[b], Fernand Léon[b] and Gerard Maze[b]

[a] Department of Mechanics, Tallinn University of Technology, Ehitajate tee 5, 19086 Tallinn, Estonia; madis.ratassepp@ttu.ee
[b] Laboratoire Ondes et Milieux Complexes, Unité Mixte de Recherche, Centre National de Recherche Scientifique 6294, Université Le Havre, Place Robert Schuman, 76600 Le Havre, France

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Abstract. Lamb modes are widely used for non-destructive evaluation of plate-like structures and simple interpretation procedures for the analysis of the monitored structures are needed. In this study we apply the orthogonality relation based method for post-processing Finite Element (FE) predictions in order to separate Lamb modes at a plate edge. The reflected wave field from the free edge is a superposition of all the eigenmodes of an infinite plate. The eigenmode amplitudes of the reflected wave field are determined by applying the orthogonality-based method. Overlapping wavepackets of Lamb modes at a plate edge are simulated by using the FE model of the incident S_0 mode in a plate with a crack. Time-domain signals of propagating and non-propagating modes are extracted.

Key words: Lamb modes, orthogonality, mode separation.

1. INTRODUCTION

Lamb modes are widely used for non-destructive evaluation of plate-like structures. Among various challenges, the separation of modes is needed for the development of a proper interpretation procedure to analyse the response of monitored structures [1]. In general it is not possible to avoid multimodality in Lamb wave testing. Even if the incident wave is a pure Lamb mode, the interaction of a wave with a defect or structural feature can result in a complicated multimode signal, since there may exist at least two propagating modes in a plate at any chosen testing frequency. It is desirable to understand how different modes are created in the structure in order to characterize the defects and structural features. More information can be obtained by analysing separated signals of different modes.

Lamb modes can be separated from a signal by applying the classical two-dimensional spatial Fourier transform technique that uses time records from a series of equally spaced points along a plate [2]. However, this technique requires long paths of the accessible plate surface to be monitored, while in the orthogonality-based method the number of measurement points of the through-thickness wave field can be significantly reduced [3]. On the other hand, in the proposed method the measurement of all displacement and stress field components simultaneously is required which currently is not possible. At the plate edge the orthogonality relation is simplified as the stresses equal to zero. Therefore only in-plain and out-of-plain displacement components have to be measured at a plate edge.

The orthogonality of the Lamb modes was shown already a long time ago [4] and this property has helped to solve several wave propagation and scattering problems in structures. The idea of using orthogonality to extract individual Lamb modes from the scattered wave fields is not new. Moreau et al. [3] used the orthogonality relation to calculate the reflection and transmission coefficient of isolated modes in case of a pure Lamb mode incident on a notch-like defect. In the paper [5] they also showed that the proposed technique can be extended to three-dimensional guided wave scattering problems in plates. The use of the modal decomposition method with the orthogonality relation has allowed to solve the Lamb wave interaction with a plate edge [6] and delaminated plate [7]. Gunawan and Hirose [8] derived a generalized orthogonality relationship for the Lamb modes of oblique scattering on the free edge of a plate. They used it to develop a mode decomposition technique for an elastodynamic field and semi-analytically obtained the reflection coefficients for the oblique incidence problem. In addition, it is important to understand the interaction of Lamb modes with a plate edge which has been studied and reported quite extensively [9–15]. It has been shown that the incident wave, interacting with a free plate edge, gives rise to a system of reflected waves, consisting of propagating and non-propagating modes. Above the cut-off frequencies of higher order modes the energy carried by the incident mode can be distributed among the other possible reflected propagating modes.

In this paper we present the orthogonality relation based method for post-processing FE predictions in order to separate Lamb modes at a plate edge in the plane strain condition. The reflected wave field from the free edge is assumed to be a superposition of all the eigenmodes of an infinite plate. The eigenmode coefficients of the reflected wave field are determined by applying the orthogonality-based method that was used to determine the reflection coefficients of Lamb modes at a plate edge [6]. Overlapping wavepackets of Lamb modes at a plate edge are simulated by using the FE model of the incident S_0 mode in a plate with a crack. Time-domain signals of several propagating and non-propagating modes are extracted.

2. ORTHOGONALITY RELATION OF LAMB MODES AT A PLATE EDGE

Figure 1 shows the two-dimensional Lamb mode, propagating towards the edge of a semi-infinite plate along the x direction. The plate medium is considered to be isotropic, homogeneous and elastic; plane strain conditions are considered.

The displacements and stresses of each Lamb mode of the order n are expressed in the vector form:

$$\boldsymbol{u}_n = \begin{pmatrix} u_x \\ u_y \end{pmatrix}_n e^{i(\omega t - k_n x)}, \tag{1}$$

$$\boldsymbol{\sigma}_n = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix}_n e^{i(\omega t - k_n x)}, \tag{2}$$

where the scalar components u_x, u_y, σ_{xx} and σ_{xy} represent the displacement and stress variation only along the y coordinate, t is time, ω is angular frequency and k_n is the complex wave number of the mode n. Detailed description is given in [16].

The total wave field, including the incident wave, can be written as a modal series of Lamb eigenmodes, which must satisfy stress-free boundary conditions on the free edge $x = 0$:

$$\begin{pmatrix} S_{xx} \\ S_{xy} \end{pmatrix} = \sum_{m=0}^{\infty} r_m \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix}_m = 0, \quad \begin{pmatrix} U_x \\ U_y \end{pmatrix} = \sum_{m=0}^{\infty} r_m \begin{pmatrix} u_x \\ u_y \end{pmatrix}_m, \tag{3}$$

where r_0 is the incident mode amplitude and r_m is the complex reflection amplitude of mode $m = 1, 2, \ldots$.

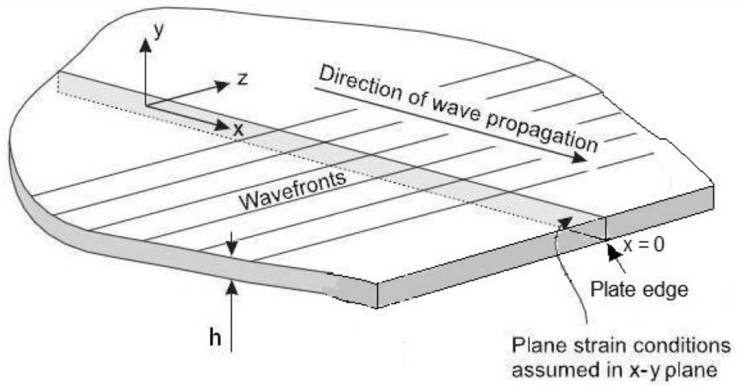

Fig. 1. Geometry of the problem.

The general orthogonality relation [3,7], which involves a scalar product of the displacement and stress distributions of two modes m and n, is considered at a given position along the plate:

$$\int_0^h [(\sigma_{xy})_n (u_y)_m + (\sigma_{xy})_m (u_y)_n - (\sigma_{xx})_n (u_x)_m - (\sigma_{xx})_m (u_x)_n] dy = a(n)\,\delta_{mn}, \quad (4)$$

where $a(n)$ is a normalization factor and δ_{mn} is the Kronecker delta symbol.

Applying the orthogonality condition to the total wave field on the free edge, complex reflection amplitudes of any mode n can be calculated knowing only the edge displacement field $(U_x; U_y)$

$$\int_0^h [(\sigma_{xy})_n (U_y)_m - (\sigma_{xx})_n (U_x)_m] dy$$
$$= \int_0^h [(\sigma_{xy})_n \sum\nolimits_m r_m (u_y)_m - (\sigma_{xx})_n \sum\nolimits_m r_m (u_x)_m] dy = a(n)\delta_{mn}, \quad (5)$$

$$a(n) = 2\int_0^h r_n [(\sigma_{xy})_n (u_y)_n - (\sigma_{xx})_n (u_x)_n] dy, \quad (6)$$

$$r_n = \frac{a(n)}{2\int_0^h [(\sigma_{xy})_n (u_y)_n - (\sigma_{xx})_n (u_x)_n] dy}. \quad (7)$$

3. POST-PROCESSING PROCEDURE FOR FE RESULTS

Figure 2 shows the FE model for Lamb modes scattered by a crack and reflected at a plate edge. Wave propagation was simulated by using finite element modelling software ANSYS [17]. A pure Lamb mode S_0 is generated on the left edge by prescribing identical displacement in x direction at all nodes on the edge. The excited mode propagates along the plate and interacts with a crack in the plate and reaches the plate edge. The interaction phenomenon causes the scattering of Lamb modes, reflected from and transmitted past the crack. Multiple reflections can take place as the crack is rather close to the plate edge. The total acoustic field in the guide can therefore be very complicated since it results from the superposition of the incident and all the scattered modes: a series of propagating modes plus an infinite number of non-propagating modes. The measured time-domain signals at the plate edge are transformed into frequency domain. As the excitation signal is chosen to be a Hanning-windowed toneburst, the extraction procedure must be performed over a range of frequencies. At each frequency step the through-thickness displacements and stresses of the separatable mode are calculated and amplitudes are predicted at the plate edge by using Eq. (7). Finally, the spectrum of the extracted mode is transformed back into time domain by using the inverse Fourier' transform.

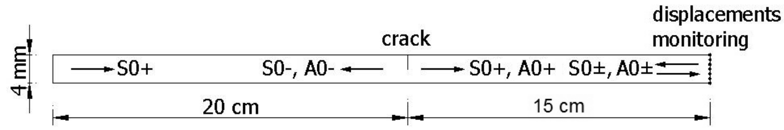

Fig. 2. FE model of S_0 mode interacting with a crack and plate edge.

4. RESULTS AND DISCUSSION

Firstly, it is important to understand the properties of the modes in a plate. Dispersion curves for the Lamb modes in a 1 mm thick aluminium plate (density $\rho = 2660$ kg/m^3, Poisson's ratio $\nu = 0.33$ and Young's modulus $E = 72.8$ GPa), including the non-propagating branches, are shown in Fig. 3. Dispersion curves

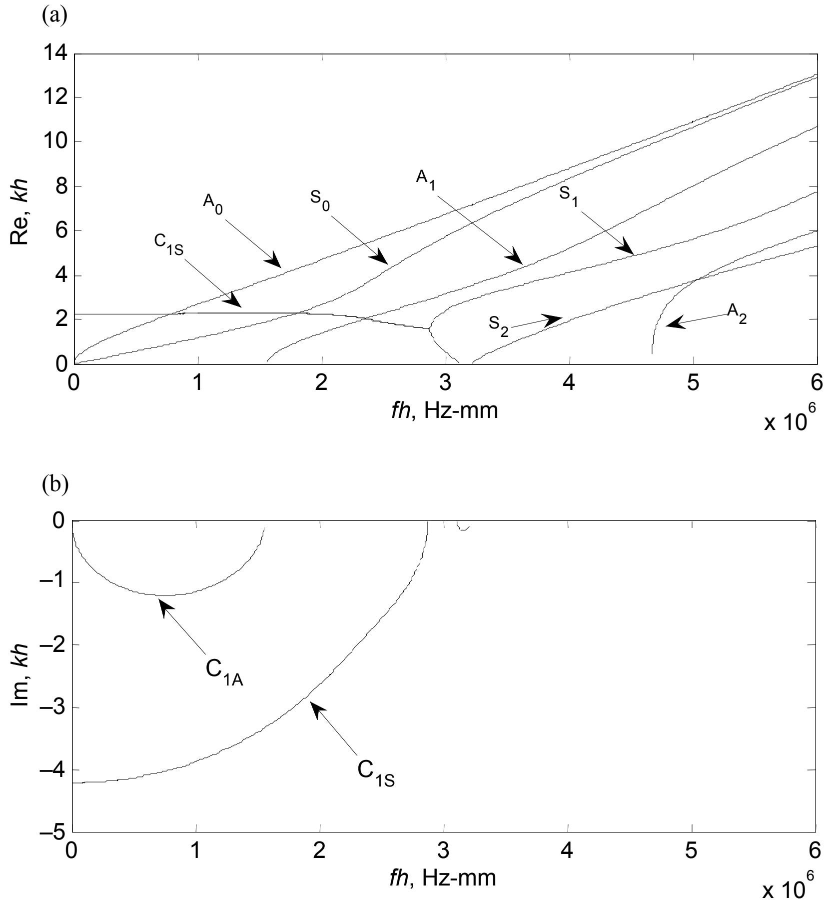

Fig. 3. Dispersion curves for Lamb modes in a 1 mm thick aluminium plate, including non-propagating branches: a) real wave number, b) imaginary wave number.

are scalable by the product of the frequency and the plate thickness which is the horizontal axis. The work presented here is focused on the S_0 and A_0 modes in the frequency-thickness range fh from 0.8 to 1.2 MHz-mm, which is below the cut-offs of higher order modes. It follows that there is no mode conversion of propagating modes at the plate edge: both modes reflect as S_0 or A_0 with the reflection amplitude of unity. The low-order non-propagating modes at the studied frequency range are C_{1A} and C_{1S}. C_{1A} is entirely imaginary and is linked to the propagating A_1 mode. C_{1S} is a complex mode which is linked to S_1 mode. Non-propagating modes do not transport energy along the plate [[9]], the imaginary part of the wavenumber gives the exponential decay of the scattered mode. It has been shown that the presence of non-propagating modes at the plate edge allows the boundary conditions to be satisfied [[9]] which in case of pure propagating modes is not possible. Therefore additional motion due to non-propagating modes at the edge has to be considered.

Secondly, finite element simulations were performed for a 4 mm thick aluminium plate where the scattering of the S_0 mode by a 2 mm deep crack was studied. The incident mode is a 10-cycle Hanning-windowed toneburst with a centre frequency 250 kHz. Figure 4 shows the simulated time domain signal of u_x displacement component, measured at the upper corner of the plate right edge. The amplitude is normalized by the peak amplitude of the incident wave packet. It can be seen that the first arrived wave packet is much wider in time than the incident pulse. It is composed of S_0 plus mode converted A_0 mode at a crack and also a number of non-propagating modes, generated at the plate edge. However, it is not possible to determine directly the amplitudes of the scattered modes from the shown signal. The influence of non-propagating modes, generated at the notch, is negligible as their amplitude decays within a distance of few plate thicknesses. For the separation of the modes the in-plane and out-of-plane displacements were monitored along the edge in 9 points with the step of 0.5 mm and the orthogonality relation was applied. Figures 5 to 9 show the time domain signal of normalized u_x displacement component of the separated S_0+, A_0+, S_0–, A_0– and the first order non-propagating mode C_{1A} of the anti-symmetric type; "+" denotes the mode propagating in positive direction and "–"

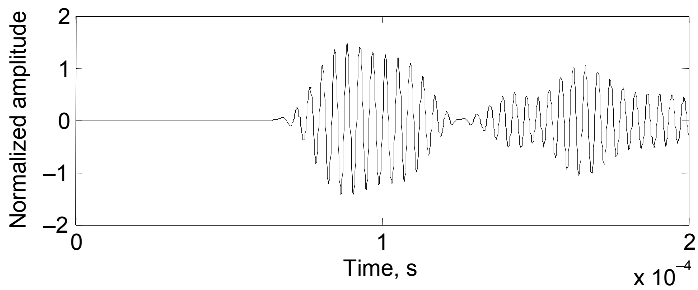

Fig. 4. *Ux* displacement of the plate edge in case of the S_0 mode incident simulated by FE modelling.

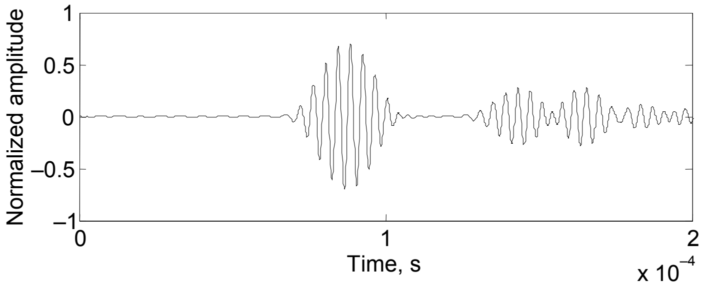

Fig. 5. U_x displacement of the separated S_0+ mode.

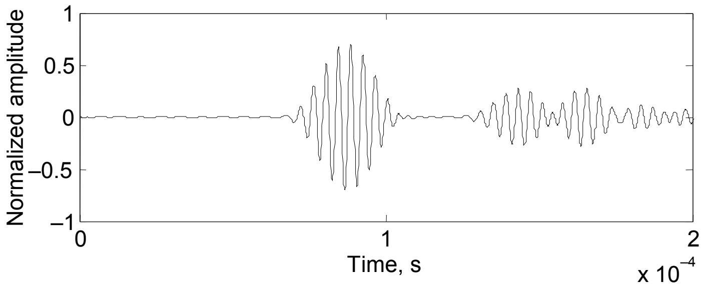

Fig. 6. U_x displacement of the separated S_0– mode.

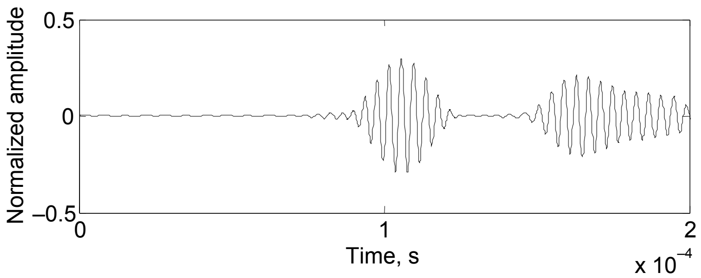

Fig. 7. U_x displacement of the separated A_0+ mode.

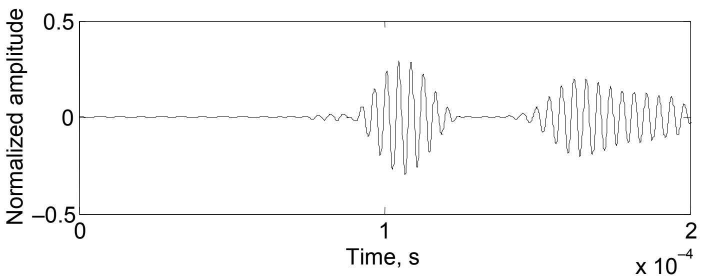

Fig. 8. U_x displacement of the separated A_0– mode.

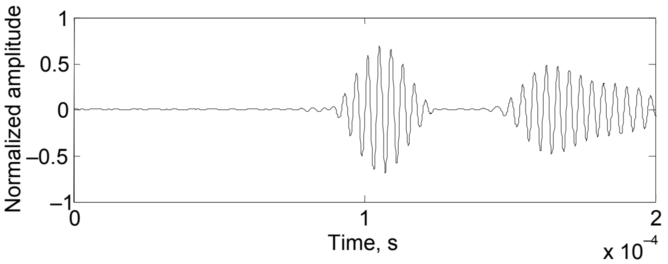

Fig. 9. U_x displacement of the separated antisymmetric C_{1A} mode.

in negative direction, respectively. The wave packets of various modes are clearly separated. The first arrival is S_0 mode, which is followed by the slower mode-converted A_0 mode, generated at the crack. The signals starting at 130 μs are the repeatedly reflected pulses from the crack and the plate edge where the incident wave was generated. Comparing the amplitudes of the waves propagating in "+" and "–" direction, it can be seen that the waves have been reflected without mode conversion at the plate edge as the amplitudes remain the same. Interestingly, the non-propagating mode C_{1A} has significant amplitude at the studied frequency range. This was observed also in [12] at the plate edge in case of incident A_0 mode. The accuracy of the results was examined by evaluating the residual stresses σ_{xx}, σ_{xy} of the total wave field at the edge. When 5 pairs of non-propagating modes are employed, the residual stresses were no more than 5% of the incident wave. Additionally, summing up the extracted signals, the exact original FE signal was obtained, which confirms the reliability of the procedure.

For practical applications it is important to investigate the errors that may occur in the extraction procedure due to the deviations in wave propagation parameters. In real measurements, the material properties of the plate are not exactly known and this may cause some deviations in through-thickness displacement and stress calculations. Another error, which may influence the procedure, is the positioning error for displacement measurements along the plate edge. The influence of the errors on the extraction procedure will be studied in a future paper.

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Lambi lainete eraldamine plaadi serval ortogonaalsustingimuste abil

Madis Ratassepp, Aleksander Klauson, Farid Chati, Fernand Léon ja Gerard Maze

Lambi laineid kasutatakse laialdaselt plaadilaadsete struktuuride mittepurustavas kontrollis. Üks oluline protseduur on plaadi lainemoodide üksteisest eraldamine, mis võimaldab arendada struktuuride seiramiseks vajalikke analüüsi- ja interpretatsioonimeetodeid. Selles uuringus kasutati lainemoodide ortogonaalsust lõplike elementide mudeli signaalide töötlemiseks Lambi lainete eraldamiseks plaadi serval. Plaadilt peegeldunud lainevälja võib kirjeldada erinevate Lambi lainete superpositsioonina, mille koefitsiente saab leida ortogonaalsustingimuste abil. Kattuvaid Lambi laineid simuleeriti praoga plaadis, kui pealeminevaks laineks oli sümmeetriline membraanlaine S_0. Meetod võimaldas eraldada plaadi serval erinevate levivate ja mittelevivate lainete signaale ajadomeenis.